Modular units and cuspidal divisor classes on $X_0(n^2M)$ with $n|24$ and M squarefree

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For a positive integer N, let $C(N)$ be the subgroup of $J_0(N)$ generated by all cuspidal divisors of degree 0 and $C(N)(\mathbb{Q}) := C(N) \cap J_0(N)(\mathbb{Q})$ be its $\mathbb{Q}$-rational subgroup. Let also $C_{\mathbb{Q}}(N)$ be the subgroup of $C(N)(\mathbb{Q})$ generated by $\mathbb{Q}$-rational cuspidal divisors. We prove that when $N = n^2M$ for some integer n dividing 24 and some squarefree integer M, the two groups $C(N)(\mathbb{Q})$ and $C_{\mathbb{Q}}(N)$ are equal. To achieve this, we show that all modular units on $X_0(N)$ on such N are products of functions of the form $\eta(m\tau + k/h)$, $mh^2|N$ and $k \in \mathbb{Z}$ and determine the necessary and sufficient conditions for products of such functions to be modular units on $X_0(N)$. This is a joint work with Liuquan Wang.

Triplet invariance under generalized inverses

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Abstract

Given a semiprime ring R, we give a complete characterization of the triplet invariance ba^-c (resp. ba^+c) under all inner inverses a^- (resp. reflexive inverses a^+) of $a \in R$. For the case of outer inverses, given a regular element a in an arbitrary ring, the outer inverses of a are completely determined. It is also proved that if R is a regular ring and $a, b, c \in R$, then the triplet $b\hat{a}c$ is invariant under all outer inverses $\hat{a}$ of a if and only if $\mathrm{E}[b]\mathrm{E}[a]\mathrm{E}[c] = 0$. Here, $\mathrm{E}[x]$ is the smallest idempotent in C, the extended centroid of R, such that $x = \mathrm{E}[x]x$. These answer the questions due to Hartwig and Patrício in 2018.

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Key words and phrases: Semiprime (prime, regular) ring, extended centroid, triplet invariance, (unit-) regular element, inner (outer, reflexive) inverse.

Speaker: Tsiu-Kwen Lee

Tilting modules for the periplectic Lie superalgebra

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The periplectic Lie superalgebra $\mathfrak{p}(n)$ is a superanalogue of the orthogonal or symplectic Lie algebra. In this talk, we will introduce a version of Ringel duality of arbitrary parabolic BGG category $\mathcal{O}$ for $\mathfrak{p}(n)$. In particular, this duality provide an approach to the problem of finding character formulae of tilting modules. This talk is based on joint works with Shun-Jen Cheng, Kevin Coulembier and Yung-Ning Peng.

Keywords: Lie superalgebras, tilting modules, character formulae.

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On function field alternating multizeta values

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In the research on number theory, it is known that there are many analogies between number fields and function fields over finite fields $\mathbb{F}_q$ (q is a power of prime number p). For example, the basic analogues are $\mathbb{F}_q[\theta]$, $\mathbb{F}_q(\theta)$, $\mathbb{F}_q((1/\theta))$ and $\mathbb{C}_\infty$ which correspond to $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ respectively. Finding such analogues is one of fundamental issues in function field number theory. In this result [H19], we introduce the *alternating multizeta values in positive characteristic* (AMZVs in short) which are generalizations of Thakur multizeta values, positive characteristic analogue of multizeta values [T04] and also function field analogues of alternating multizeta values (for the detail of number fields case, see [Z16]). The AMZVs are defined as the following infinite sums ([H19]):

For $\mathfrak{s} = (s_1, \ldots, s_r) \in \mathbb{N}^r$ and $\boldsymbol{\epsilon} = (\epsilon_1, \ldots, \epsilon_r) \in (\mathbb{F}_q^\times)^r$,

$$\zeta(\mathfrak{s}; \boldsymbol{\epsilon}) = \sum_{\substack{a_1,\ldots,a_r \in \mathbb{F}_q[\theta] \\ a_1,\ldots,a_r:\text{monic} \\ \deg a_1 > \cdots > \deg a_r \geq 0}} \frac{\epsilon_1^{\deg a_1} \cdots \epsilon_r^{\deg a_r}}{a_1^{s_1} \cdots a_r^{s_r}} \in \mathbb{F}_q((1/\theta)).$$

We call $\mathrm{wt}(\mathfrak{s}) := \sum_{i=1}^r s_i$ the weight and $\mathrm{dep}(\mathfrak{s}) := r$ the depth of the presentation of $\zeta(\mathfrak{s}; \boldsymbol{\epsilon})$. We also show their fundamental properties listed in below.

(A). Non-vanishing ([H19, Theorem 2.1]) For any $\mathfrak{s} = (s_1, \ldots, s_r) \in \mathbb{N}^r$ and $\boldsymbol{\epsilon} = (\epsilon_1, \ldots, \epsilon_r) \in (\mathbb{F}_q^\times)^r$, $\zeta(\mathfrak{s}; \boldsymbol{\epsilon})$ are non-vanishing.

(B). Sum-shuffle relations ([H19, Theorem 2.6]) For $\mathfrak{a} := (a_1, a_2, \ldots, a_r) \in \mathbb{N}^r$, $\mathfrak{b} := (b_1, b_2, \ldots, b_s) \in \mathbb{N}^s$, $\boldsymbol{\epsilon} := (\epsilon_1, \epsilon_2, \ldots, \epsilon_r) \in (\mathbb{F}_q^\times)^r$ and $\boldsymbol{\lambda} := (\lambda_1, \lambda_2, \ldots, \lambda_s) \in (\mathbb{F}_q^\times)^s$, we may express the product $\zeta(\mathfrak{a}; \boldsymbol{\epsilon})\zeta(\mathfrak{b}; \boldsymbol{\lambda})$ as follows:

$$\zeta(\mathfrak{a}; \boldsymbol{\epsilon})\zeta(\mathfrak{b}; \boldsymbol{\lambda}) = \sum_i f_i'' \zeta(c_{i1}, \ldots, c_{il_i}; \mu_{i1}, \ldots, \mu_{il_i})$$

for some $c_{ij} \in \mathbb{N}$ and $\mu_{ij} \in \mathbb{F}_q^\times$ so that $\sum_{m=1}^r a_m + \sum_{n=1}^s b_n = \sum_{h=1}^{l_i} c_{ih}$, $\prod_{m=1}^r \epsilon_m \prod_{n=1}^s \lambda_n = \prod_{h=1}^{l_i} \mu_{ih}$, $l_i \leq r + s$ and $f_i'' \in \mathbb{F}_p$ for each i.

(C). Period interpretation ([H19, Theorem 3.4]) For $\mathfrak{s} = (s_1, \ldots, s_r) \in \mathbb{N}^r$ and $\boldsymbol{\epsilon} = (\epsilon_1, \ldots, \epsilon_r) \in (\mathbb{F}_q^\times)^r$, $\zeta(\mathfrak{s}; \boldsymbol{\epsilon})$ are periods of pre-t-motive M defined in [H19] Definition 3.2.

(D). Linear independence ([H19, Theorem 4.7]) Let $w_1, \ldots, w_l \in \mathbb{N}$ be distinct. We suppose that V_i is a $\mathbb{F}_q(\theta)$-linearly independent subset of AZ_{w_i} for $i = 1, \ldots, l$. Then the following union

$$\{1\} \bigcup_{i=1}^{l} V_i$$

is a linearly independent set over $\overline{\mathbb{F}_q(\theta)}$ (a fixed algebraic closure of $\mathbb{F}_q(\theta)$ in $\mathbb{C}_\infty$), that is, there are no nontrivial $\overline{\mathbb{F}_q(\theta)}$-linear relation among elements of $\{1\} \bigcup_{i=1}^{l} V_i$.

Here we denote AZ_w the set of monomials of AMZVs with total weight w. Further, the total weight is defined for the monomial $\zeta(\mathfrak{s}_1; \boldsymbol{\epsilon}_1)^{m_1} \cdots \zeta(\mathfrak{s}_n; \boldsymbol{\epsilon}_n)^{m_n}$ as

$$\sum_{i=1}^{n} m_i w_i$$

where $m_1, \ldots, m_n \in \mathbb{Z}_{\geq 0}$ which are not all zero and $\zeta(\mathfrak{s}_1; \boldsymbol{\epsilon}_1), \ldots, \zeta(\mathfrak{s}_n; \boldsymbol{\epsilon}_n)$ be AMZVs of $\mathrm{wt}(\mathfrak{s}_i) = w_i$ $(i = 1, \ldots, n)$.

For the property (A), it is immediately obtained by an inequality property of the absolute values of power sums proved by Thakur [T09]. For the property (B), we use Chen's formula [Ch15] and approach the higher depth case by induction method invented by Thakur [T10]. This enable AMZVs to form an $\mathbb{F}_p$-algebra. For the property (C), inspired by [AT09] and Anderson-Thakur polynomials ([AT90]) that can interpolate power sums, we use those polynomials to create suitable power series that their specialization are AMZVs and then we use these series to create suitable pre-t-motives to establish the period interpretation of (C). By this property, we can proceed to the property (D). For the linear independence property of AMZVs (D), we modify the method [C14] by applying Anderson-Brownawell-Papanikolas criterion [ABP04] to establish the alternating analogue of MZ property for AMZVs. By the property (D) ,we can show that there are no $\overline{\mathbb{F}_q(\theta)}$-linear relation between AMZVs of different weights and that each AMZV $\zeta(\mathfrak{s}; \boldsymbol{\epsilon})$ is transcendental over $\overline{\mathbb{F}_q(\theta)}$.

Keywords: multizeta values in positive characteristic, non-vanishing, sum-shuffle relation, pre-t-motive, linear independence

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Characterization of Zeta functions in differential equations and dynamical systems

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Traditional efforts on proving Riemann Hypothesis have been focusing on analytical, algebraic and algebraic geometrical aspects. Recently, computing efforts have extended to assert that ~10^{30} zeta zeros are locating on critical line. Nevertheless, the mathematical communities are still expecting that the continuous studies may also produce impacts on other branches of mathematics as well as promote broad applications in different scientific and technological fields.

In ICM 2014, we proposed that Cauchy-Riemann equations of zeta functions in conjunction with Bézout theorem as follows:

- $\{s := Real(\zeta'(s)) = 0\} \cap \{s' := Imag(\zeta'(s)) = 0\}$
  $= \{(0.5 + i * t_1), (\sigma_t + i * t_2)\}$
- $\sigma_t \in \mathbb{R}$
- $\zeta(0.5 + i * t_1) = 0$
- $\zeta(\sigma_t + i * t_2)$ *is a saddle point*

In ICM 2018, we further investigated critical strip in the context of dynamical systems. We presented that the isolation of individual non-trivial zeta zeros by the divergent regions asserts Montgomery's Pair Correlation Conjecture and further provides global-local connections of zeta zeros for constructing the structure for the proof of Riemann Hypothesis.

In this presentation, we propose the strategic path from the establishments of proofs of Riemann Hypothesis in finite field to infinite field together with isolation of zeta zeros, global-local connections, and Cauchy-Riemann equations of zeta functions.

Keywords: Riemann Hypothesis, Zeta Function, Cauchy-Riemann equation, dynamical systems.

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Automorphism groups of holomorphic vertex operator algebras of central charge 24

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In this talk, we will first discuss the full automorphism group of a certain orbifold VOA $V_{\Lambda_g}^g$ associated with a coinvariant lattice Λ_g of the Leech lattice. We will also discuss how to use the information of $\mathrm{Aut}\,(V_{\Lambda_g}^g)$ to determine the full automorphism groups of several holomorphic VOAs of central charge 24.

This is a joint work with K. Betsumiya and H. Shimakura.

Class Number Relations Arising From Intersections Of Shimura Curves And Humbert Surfaces

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Abstract

In this talk, I will present coefficients of Cohens Eisenstein series of weight 5/2 as some class number relations. These relations arise from the intersections of Shimura curves and Humbert surfaces on the Siegel modular threefold, and can be considered as a higher-dimensional analogue of the classical Hurwitz-Kronecker class number relation.

This is a joint work with Yifan Yang.

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ON THE ALGEBRAICITY OF THE SYMMETRIC SIXTH POWER L-FUNCTIONS OF ELLIPTIC MODULAR FORMS

SHIH-YU CHEN

Abstract. Deligne's conjecture predicted that the period for the critical values of a symmetric even power L-function of an elliptic modular form is equal to a power of the Petersson norm of the associated modular form. In this talk, we present our result on the algebraicity of the symmetric sixth power L-functions of elliptic modular forms. We show that the period in this case is equal to product of Petersson norms of the associated modular form and certain holomorphic Siegel modular form of degree two. Our result is compatible with Deligne's conjecture.